

GEO-PHYSICS
Paper I

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **TEN** questions divided under **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Questions No. 1 and 6 are **compulsory**.

Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION 'A'

- 1.(a) Calculate the present-day relative motion at location 22°S and 25°E in the Botswana using India-Eurasia rotation pole at 24°N and 18°E. Assume the radius of the Earth to be 6371 km and angular velocity is 7.2×10^{-7} degree/year. 10

- 1.(b)(i) An earthquake is generated by a rupture along a fault having a length of 100 km, width of 20 km and an average displacement of 5 m. Assuming the average modulus of rigidity of the earth's crust to be 30 GPa, calculate the magnetic moment (M_w) of the earthquake. 5

- 1.(b)(ii) A continental crust is modeled as a homogeneous, isotropic elastic medium with Young's modulus $E = 75$ GPa, Poisson's ratio $\sigma = 0.26$ and density $\rho = 2.8$ g/cc. Determine the shear and bulk moduli and calculate the P- and S- wave velocities in the medium. 5

- 1.(c) For a given vector field

$$\vec{F} = (x^2 yz)\hat{i} + (xy^2 z)\hat{j} + (xyz^2)\hat{k},$$

- (i) Find the divergence of $\vec{F}$
- (ii) Evaluate $\nabla \cdot \vec{F}$ at point (1, 1, 1)
- (iii) Find the curl of $\vec{F}$ and comment whether the field is irrotational.

4+2+4=10

- 1.(d) Differentiate between Forward and Inverse problems. Explain direct and model based inversion techniques with suitable geophysical examples. 10

- 2.(a) Derive expression for the Geoid height anomaly over a mountain range for Airy compensation model.

Assuming a constant continental crustal thickness of 35 km, calculate the geoid height anomaly due to a mountain of height 4 km for Airy-type compensation.

Given : Gravitational constant, $G = 6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \cdot \text{s}^{-2}$

Acceleration due to gravity, $g = 9.8 \text{ ms}^{-2}$

Radius of the Earth, $R = 6371 \text{ km}$

Crustal density = 2.8 g/cc

Mantle density = 3.3 g/cc

5+5=10

- 2.(b) Explain the concept of seafloor spreading and critically discuss the geophysical evidences supporting it. 10
- 2.(c) A tectonic region follows the Gutenberg-Richter frequency-magnitude relationship with a b-value of 0.9.
- (i) Estimate the ratio of annual occurrence rates of earthquakes with magnitudes $M \geq 5.5$ and $M \geq 7.0$
- (ii) Calculate the total annual seismic energy released by earthquakes with magnitudes between 5.5 and 7.0. 5+5=10
- 3.(a) Assume that the Earth is spherically symmetric and composed of concentric homogeneous shells with no chemical and phase changes within shells. Pressure and density are assumed to increase purely hydrostatically. Derive the Adams-Williamson equation for the radial variation of the density in the Earth and discuss it with suitable figure. 10
- 3.(b) A seismic refraction survey is conducted over a homogeneous horizontally layered Earth. In one model, the crust is 35 km thick, with P-wave velocity of 5.8 km/s, overlying an upper mantle with P-wave velocity of 8.0 km/s. In a second model, the crust consists of an upper layer 25 km thick with a P-wave velocity of 5.8 km/s, underlain by a 10 km thick low-velocity layer with a P-wave velocity of 4.5 km/s; the mantle velocity remains 8.0 km/s.
- Plot the first-arrival travel-time curves for both models on a single distance – time diagram and discuss the effect of the low-velocity layer on the estimation of the depth to the top of the mantle. 10
- 3.(c) The data obtained from a velocity reflection survey are $t_{0,1} = 6.0$ s, $t_{0,2} = 15.0$ s, $\tilde{\alpha}_1 = 6.0$ km/s and $\tilde{\alpha}_2 = 7.0$ km/s where $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ represent the time-weighted root-mean-square velocity at first and second interfaces, respectively and $t_{0,1}$ and $t_{0,2}$ are the normal incidence two-way travel time, respectively. Determine the velocity-depth model for the survey, assuming that each subsurface layer has a constant seismic velocity. 10

- 4.(a) During a geophysical survey, charge accumulation is observed on two underground mineral bodies due to electrostatic induction caused by tectonic stress. The two bodies can be approximated as point charges.

Charge on Body A, $q_1 = +5 \times 10^{-6} \text{C}$
Body B, $q_2 = -8 \times 10^{-6} \text{C}$

Distance (r) between A and B = 0.40 m

Relative permittivity of surrounding rock medium $\epsilon_r = 4$.

- (i) Calculate the electrostatic force between the two bodies.
- (ii) State whether the force is attractive or repulsive.
- (iii) Briefly comment on the geophysical significance of such electrostatic forces in subsurface exploration.

Take : $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$.

5+2+3=10

- 4.(b) (A) Describe the geomagnetic field of earth and its components. Establish the relationship between the various components.
- (B) At a place on Earth's surface, the angle of dip is 30° and total magnetic field is $B = 5 \times 10^{-5} \text{ T}$

Calculate :

- (i) the horizontal component H
 - (ii) the vertical component V
- (C) Also, describe how it protects the Earth from solar and cosmic radiation.

4+4+2=10

- 4.(c) Define Model Resolution matrix. Explain its importance and compute the model Resolution matrix for the following Kernel matrix :

$$G = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

10

- 5.(a) For a given charge and current distribution, the scalar potential V and the vector potential A related to the electric field vector E and magnetic field vector B by the equations :

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$B = \nabla \times A$$

Rewrite Maxwell's equations in terms of these potentials for linear, homogenous media. 10

5.(b) Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

Find singular value decomposition (SVD) of matrix A i.e. $A = U\Sigma V^T$ where U and V are orthogonal matrix, Σ is a diagonal matrix of singular values. Also using SVD, determine the rank and 2-norm of A . 10

- 5.(c) In geophysics, inversion problem often involve finding a model m (e.g. subsurface velocity, density) that minimizes the misfit function

$$\phi(m) = \|d_{obs} - G(m)\|^2$$

where d_{obs} is observed data

$G(m)$ is forward operator

- (A) Explain why Genetic Algorithms (GA) are particularly suitable for geophysical inverse problems highlighting advantages over gradient based methods.
- (B) Discuss the theoretical limitations of GA or Simulated Annealing (SA) when applied to geophysical inverse problems, particularly regarding ill-posedness and non-uniqueness. 5+5=10

SECTION 'B'

- 6.(a) Prove that for a laminar flow of fluid or field, the equation of continuity can be written as $\text{div } \vec{V} = 0$. 10
- 6.(b) Explain Ampere's Circuital law in magnetostatics. Elaborate the steps adopted by Maxwell in modifying this equation for time dependent fields. 10
- 6.(c) With schematic diagrams please explain how transmission lines are used for
- (i) Load matching
 - (ii) Impedance measurements 10

- 6.(d) (i) Explain different system segments of Global Positioning System (GPS). 5
- (ii) Calculate range error in GPS L_1 signal due to presence of ionosphere having total electron contents (TEC) of 15 TECU. (Here frequency of L_1 signal, $f = 1575.42$ MHz) (1 TECU = 1×10^{16} ele/m³) 5

- 7.(a) Assuming that there is no singularity in finite plane of differential equation

$$y'' - 2xy' + 2\lambda y = 0 \quad \text{where } \lambda = \text{constant}$$

prove that its solution is Hermite's polynomial of order n . 10

- 7.(b) What is Green's function ? Describe its importance alongwith its properties. 10

- 7.(c)(i) Explain how a definite Eulerian integral is another form of Beta function. 4

- 7.(c)(ii) Prove that Beta and Gamma functions are related to each other as below :

$$\beta(m, n) = \frac{\Gamma(n) \Gamma(m)}{\Gamma(m+n)} \quad 6$$

- 8.(a) Using Gauss's law, obtain Poisson's equation for a scalar field. Under what conditions does Poisson's equation become identical to Laplace equation ? 10

- 8.(b) Given that $\vec{D} = z\rho \cos^2\phi \hat{a}_z$ C/m², calculate the charge density at $(1, \pi/4, 3)$ and the total charge enclosed by the cylinder of radius 1 m with $-2 \leq z \leq 2$ m. 10

- 8.(c) (i) Explain briefly the different types of magnetohydrodynamic (MHD) waves. 5

- (ii) Show that the velocity of magnetosonic wave, V_M is related with the Alfvén velocity, V_A and sound velocity, V_S by the relation : $V_M^2 = V_S^2 + V_A^2$. 5

- 9.(a) Using the plane wave propagation in conducting medium prove that the multi-strand wires are better conductor of AC current than a single core wire of same area of cross-section. 10

9.(b) A rectangular waveguide of dimensions $a = 2$ cm, $b = 1$ cm operates below 14 GHz. How many TE and TM modes can the waveguide use if it is filled with a medium of $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu_r = 1$. Also calculate the cutoff frequencies of modes. 10

9.(c) Using Maxwell's equations for reflection of plane wave at an oblique angle prove that there are two types of polarizations taking place (i) Parallel polarization (ii) Perpendicular polarization and the equations for them is Fresnel equation. 10

10.(a) Derive the Hydrostatic equation for the atmosphere in a hydrostatic balance :

$$\frac{\partial p}{\partial z} = -g\rho$$

Where ρ is the density of air, p is pressure of the air and g is the gravitational acceleration. 10

10.(b) Describe basic chemical processes for formation of ionosphere. Explain with proper reactions, why NO^+ is a dominant ion in E-region and O^+ is in F-region ? 10

10.(c) Explain different types of ionospheric irregularities and differentiate between Sporadic-E and Spread-F irregularities. 10

